5th international workshop on ζ functions in algebra and geometry

Université Nice Côte d'Azur, May 2nd-6th 2022

Abstracts

Josep Alvarez Montaner

Title: Bernstein-Sato polynomials in singular varieties.

Abstract: In this talk we will review some recent development on the theory of Bernstein-Sato polynomials and related D-module theoretic invariants for direct summands of polynomial rings. This is a class of singular varieties including toric varieties and rings of invariants of finite groups.

- Based on joint works with C. Huneke, D. Hernández, J. Jeffries, L. Núñez-Betancourt, P. Teixeira and E.Witt.

Margaret Bilu

Title: Zeta statistics.

Abstract: Many questions in number theory have a natural analogue, of more geometric nature, formulated in the Grothendieck ring of varieties. For example, Poonen's finite field Bertini theorem has a motivic counterpart due to Vakil and Wood; however, despite the clear similarities between these two results, none of the two can be deduced from the other. The aim of this talk is to describe and motivate a conjectural way of comparing such statements in arithmetic and motivic statistics, by reformulating them in terms of the convergence of zeta functions in different topologies. We will finish by mentioning some concrete settings where our conjectures are satisfied.

- This is joint work with Ronno Das and Sean Howe.

Nero Budur

Title: Representation zeta function of $SL_n(\mathbb{Z})$.

Abstract: Using jet spaces we show that the space of representations of the fundamental group of a compact Riemann surface of genus > 1 has rational singularities. We apply this to the representation zeta function of $SL_n(\mathbb{Z})$ and bound the growth of the number of irreducible complex representations of $SL_n(\mathbb{Z})$ of dimension m for fixed $n > 2$.

Jean-Baptiste Campesato

Title: Motivic, logarithmic, and topological Milnor fibrations.

Abstract: We compare the topological Milnor fibration and the motivic Milnor fibre of a regular complex function with only normal crossing singularities by introducing their common extension: the complete Milnor fibration for which we give two equivalent constructions. The first one extends the classical Kato-Nakayama log-space, and the second one, more geometric, is based on a the real oriented version of the deformation to the normal cone. During this talk I will mainly present the first approach. In particular, we recover the topological Milnor fibration by quotienting the motivic Milnor fibration with suitable powers of $(0, +\infty)$. Conversely, we also show that the stratified topological Milnor fibration determines the classical motivic Milnor fibre.

-(joint work with Goulwen Fichou and Adam Parusiński).

Angela Carnevale

Title: Growth in nilpotent Lie rings.

Abstract: Generalised Igusa functions are combinatorially-defined rational functions. They can be used to provide formulae for ideal zeta functions of large families of class-2 nilpotent Lie rings. We will see how computations involving generalised Igusa functions unify and generalise numerous previous results pertaining to zeta functions of groups and rings. In my talk, I will put a special emphasis on combinatorial aspects and related ordered structures.

-This is based on joint work with Michael Schein and Christopher Voll.

Wouter Castryck

Title: On the scrollar invariants of a $\mathbb{P}^1$-cover and its resolvents.

Abstract: The scrollar invariants (or Maroni invariants) of a cover $\varphi : C \to \mathbb{P}^1$ describe the splitting type of $\mathcal{O}_C$ when pushed-forward to $\mathbb{P}^1$. It was observed by Casnati that if φ has degree 4, then the scrollar invariants of its cubic resolvent are essentially equal to Schreyer's invariants b_1, b_2 that show up when studying the relative canonical syzygies of φ. In this talk I will discuss joint work with Floris Vermeulen and Yongqiang Zhao, in which we generalize this observation to arbitrary degrees and arbitrary resolvents. As a by-product, we will see that "Gassmann equivalent" function fields have the same scrollar invariants. The number-theoretic counterpart of this statement would read that arithmetically equivalent number fields (by which we mean number fields having the same Dedekind zeta function) have rings of integers whose corresponding Minkowski lattices have roughly the same successive minima.

Raf Cluckers

Title: A partial overview of motivic and uniform p-adic integration.

Abstract: I will give a partial overview on the theory of motivic integration, with a focus on uniform p-adic integration. I will also describe some open questions and challenges about descent, that is, when passing from a larger p-adic field to a p-adic subfield.

Alexander Esterov

Title: The monodromy conjecture, Newton polytopes and tropical geometry.

Abstract: In a narrow sense, I will outline the proof of the monodromy conjecture for generic singularities of functions of four variables with a prescribed Newton polyhedron.

In a broader sense, I will discuss several new phenomena that become visible starting from this generality, and some new methods to deal with these phenomena.

For instance, starting from dimension 4, the monodromy conjecture for a generic function F with a prescribed Newton polyhedron N is not easily reduced to a combinatorial statement about N. This is because a lot is known in terms of N about the monodromy of the singularity of F at 0, but not at nearby points. I will fill this gap (not only in the context of Newton polyhedra) using some tropical geometry.

-The talk is based on a joint work with A. Lemahieu and K. Takeuchi.

Arthur Forey

Title: Equidistribution of exponential sums over finite fields.

Abstract: Many exponential sums over finite fields, such as Gauss or Salié-Kloosterman sums, appear as the Fourier-Melin transform of the trace function of an l-adic sheaf on a commutative algebraic group. We are interested in the equidistribution of such sums as the character varies. Generalizing work by Deligne and Katz in the cases of additive and multiplicative groups, a Tannakian formalism always controls the equidistribution. The limit measure can then be explicitly computed in some cases by exploiting analytic properties of the L-function of the Fourier-Melin transform. This is a collaboration with Javier Fresán and Emmanuel Kowalski.

Jean-Michel Granger

Title: Laplace type integrals for Gevrey series solutions of irregular hypergeometric systems.

Abstract: Let A be a full rank $d \times n$ matrix, with integer coefficients, and $d < n$. Let $\beta \in \mathbb{C}^n$. Gelfand, Kapranov and Zelevinski define differential systems $\mathcal{M}(A, \beta)$ depending on these data, commonly known as GKZ systems. Such systems are holonomic with a fixed holonomic rank for generic values of β. They are natural when we consider the columns of A as characters on the torus $(\mathbb{C}^*)^d$, inducing a diagonal action on $\mathbb{C}^n$.

The (ir)regularity of a GKZ system is equivalent to a (non)homogeneity condition. Esterov and Takeuchi proved that when β is generic, Laplace type integral representations, yield a basis of the solutions of $\mathcal{M}(A, \beta)$.

Explicit bases of the space of formal Gevrey solutions along the singular locus are known. We address the problem of representing all these solutions as asymptotic expansions of integrals along rapid decay cycles in the sense of Marco Hien. We give a complete answer to this problem for uniline matrix A, and for higher d under certain conditions on β and A. I will discuss various attempts to lighten the condition on the matrix A.

- This talk is based on papers in common with F.J. Castro-Jimenez, and M.C. Fernandez-Fernandez.

Immanuel Halupczok

Title: Canonical stratifications.

Abstract: In this talk, I will present a notion of canonical stratifications, which exist in a variety of contexts (e.g. algebraic sets over $\mathbb{C}$, semi-algebraic sets over $\mathbb{R}$, sub-analytic sets) and which have stronger regularity properties than Whitney stratifications. In particular, the local motivic Poincaré series at a point of some stratum is trivial along that stratum.

- This is joint work with David Bradley-Williams.

Sabir Gusein-Zade

Title: Weil-Poincaré series of collections of surface valuations and topology.

Abstract: It was shown that for a collection of curve valuations on $(\mathbb{C}^2, 0)$ and also on $(E_8, 0)$ (where E_8 is the surface singularity of the E_8-type) the Poincaré series coincides with the Alexander polynomial of the corresponding algebraic link. Moreover, for the plane case $(\mathbb{C}^2, 0)$ the Poincaré series determines the minimal embedded resolution of the curve (and thus the topology of the link), whereas for the E_8-singularity one has few explicitely described exceptions. In the discussed cases the Alexander polynomial can be expressed as an integral with respect to the Euler characteristic over the space of divisors on the singularity. The coincidence of the Poincaré series with the Alexander polynomial is related with the fact that on $(\mathbb{C}^2, 0)$ and $(E_8, 0)$ all divisors are Cartier. We define a natural generalization of the Alexander polynomial of an algebraic link on other surface singularity (the Weil-Poincaré series) as the integral over the space of Weil divisors. The Weil-Poincaré series is a power series with rational exponents. We discuss to which extent the Weil-Poincaré series determines the topology of the curve for rational double point surface singularities. We give analogous statements for collections of divisorial valuations.

-The talk is based on a joint work with A. Campillo and F. Delgado.

Ming-Hsuan Kang

Title: Zeta functions of complexes arising from $PGL(n)$.

Abstract: We introduce zeta functions of complexes arising from $PGL(n)$, which are high dimensional analogs of Ihara zeta functions. For all $1 \leq k \leq n-1$, there would be a zeta function that counts the k-dimensional closed combinatorial geodesics on the complex. We will show that these combinatorial geodesics have nice geometric properties so that they are the canonical generalizations of geodesics on graphs. Besides, using a cohomological approach, we establish an identity involving the alternating product of zeta functions of all dimensions and the Langlands L-function of the complex.

Winnie Li

Title: An application of zeta functions: distribution of primes.

Abstract: As a counting function, the zeta function often can be expressed as a product over primes. In this talk we shall explain, from concrete examples, how to obtain information on the distribution of primes from the analytic behavior of the zeta function.

Luis Narváez

Title: On quasi free structures.

Abstract: For a hypersurface singularity, the property of being free has been shown very useful for the description of its D-module theoretical invariants. In this talk we will recall what a quasi-free structure on a hypersurface is (Castro and Ucha, 2004), we will review on some interesting families of examples - such as prehomogeneous determinants, multifree hyperplane arrangements and many others -, and we will explain how many results about logarithmic-meromorphic comparison on free divisors can be extended to the quasi-free case.

- This is a common work with F. J. Castro Jiménez and D. Mond.

Naud Potemans

Title: The Dlt Motivic Zeta Function.

Abstract: In this talk, I will present the dlt motivic zeta function that was introduced by Xu. This adaptation of the classical motivic zeta function is defined on a dlt modification via a Denef-Loeser-like formula, replacing classes of strata in the Grothendieck ring of varieties by stringy motives. Contrary to what Xu originally claimed, the dlt motivic zeta function depends on the choice of dlt modification, and it is therefore not well-defined.

- This is joint work with Johannes Nicaise and Wim Veys.

Tobias Rossmann

Title: Zeta functions enumerating linear orbits and conjugacy classes.

Abstract: I will give an overview of recent work on zeta functions enumerating linear orbits and conjugacy classes of suitable groups. The groups in question are obtained from unipotent group schemes by taking rational points over quotients of compact discrete valuation rings. A central theme is to provide conceptual explanations for uniformity phenomena. This relies on a blend of techniques from algebra, combinatorics, and p-adic integration.

- Parts of this talk are based on joint projects with Angela Carnevale and Christopher Voll, respectively.

Christian Sevenheck

Title: Free divisors, V-filtration and Hodge ideals.

Abstract: I will describe some results (joint work with Alberto Castaño Dominguez and Luis Narváez Macarro) on Hodge ideals for a specific class of divisors with non-isolated singularities. Hodge ideals as defined by Mustaţă and Popa generalize multiplier ideals and are given by the Hodge filtration on the module of meromorphic functions along a divisor. However, they are usually hard to determine. For a certain class of free divisors (e.g. free hyperplane arrangements), one can rely on specific symmetry properties of the Bernstein-Sato polynomial of the divisor, and a basic property of Hodge modules, called strict specializability, to give a purely algebraic description of all Hodge ideals. I will explain this approach, and discuss some significant examples.

Jeremy Usatine

Title: Motivic integration for Artin stacks.

Abstract: A standard method for studying a singular variety is to resolve it by a smooth variety and to then relate invariants of the singular variety to invariants of the smooth one. Motivic integration provides powerful tools for obtaining such a relationship, especially when the invariants of interest can be described by Igusa-type integrals. Motivated by the McKay correspondence, I will describe a context in which interesting varieties admit natural resolutions of singularities by Artin stacks. This suggests a need for versatile tools in studying these "stacky" resolutions of singularities. I will discuss joint work with M. Satriano in which we use motivic integration to provide such tools, and I will also explain how our work leads to a notion of crepantness for stacky resolutions of singularities.

Marlies Vantomme

Title: Graded submodule zeta functions of pattern algebras.

Abstract: (Graded) Submodule zeta functions are associated to an algebra of endomorphisms of a (graded) module. Pattern algebras are examples of such algebras of endomorphisms. In this talk, I will introduce these terms and discuss some properties of the (graded) submodule zeta functions associated to pattern algebras.

Yimu Yin

Title: Motivic zeta function and the Hrushovski-Kazhdan style integration in valued fields.

Abstract: I will describe, after the seminal work of Hrushovski and Loeser, how to formulate the motivic zeta function attached to a complex regular function in terms of Hrushovski-Kazhdan integration in algebraically closed valued fields. In particular, I will explain the role played by an object called 'the nonarchimedean Milnor fiber', which may be considered, by way of model theory, as an infinitesimal limit of the usual topological Milnor fiber. It enables a more direct computation of motivic Milnor fiber using the Hrushovski-Kazhdan integral. I will also present some further work along this line.

- (joint with Goulwen Fichou and Arthur Forey).

Federico Zerbini

Title: Number theoretical aspects of Feynman amplitudes.

Abstract: Scattering amplitudes compute the probability of interaction of elementary particles. Feynman's perturbative approach consists in approximating them with an asymptotic series, whose coefficients are computed by Feynman integrals. These integrals can be expressed in terms of interesting periods, such as multiple zeta values and L-values of modular forms. On the one hand, this led to applying powerful cohomological tools to study amplitudes. On the other hand, new classes of periods arising from amplitudes have inspired new exciting research lines in mathematics. After an overview on Feynman amplitudes and periods, I will focus on string theory amplitudes, reporting on their relation with Brown's theories of single-valued periods and of multiple modular values.